3 | INTERFERENCE

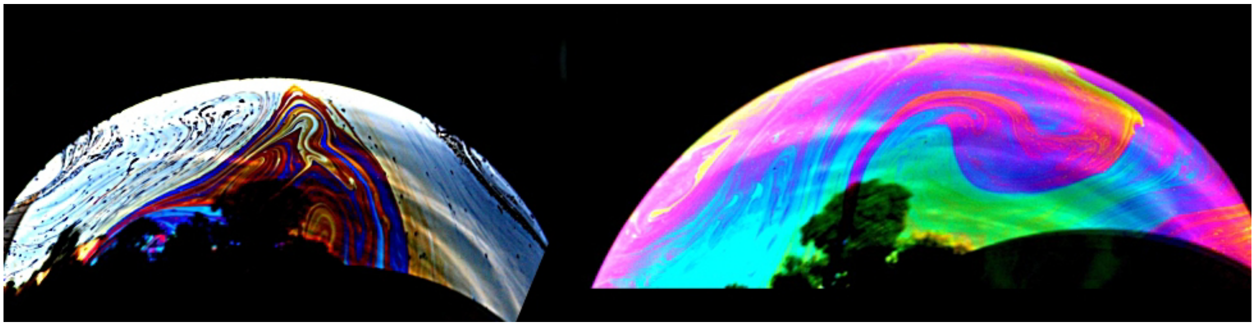


Figure 3.1 Soap bubbles are blown from clear fluid into very thin films. The colors we see are not due to any pigmentation but are the result of light interference, which enhances specific wavelengths for a given thickness of the film.

Chapter Outline

3.1 Young's Double-Slit Interference
3.2 Mathematics of Interference
3.3 Multiple-Slit Interference
3.4 Interference in Thin Films
3.5 The Michelson Interferometer

Introduction

The most certain indication of a wave is interference. This wave characteristic is most prominent when the wave interacts with an object that is not large compared with the wavelength. Interference is observed for water waves, sound waves, light waves, and, in fact, all types of waves.

If you have ever looked at the reds, blues, and greens in a sunlit soap bubble and wondered how straw-colored soapy water could produce them, you have hit upon one of the many phenomena that can only be explained by the wave character of light (see **Figure 3.1**). The same is true for the colors seen in an oil slick or in the light reflected from a DVD disc. These and other interesting phenomena cannot be explained fully by geometric optics. In these cases, light interacts with objects and exhibits wave characteristics. The branch of optics that considers the behavior of light when it exhibits wave characteristics is called wave optics (sometimes called physical optics). It is the topic of this chapter.

3.1 | Young's Double-Slit Interference

Learning Objectives

By the end of this section, you will be able to:

- Explain the phenomenon of interference
- Define constructive and destructive interference for a double slit

The Dutch physicist Christiaan Huygens (1629–1695) thought that light was a wave, but Isaac Newton did not. Newton thought that there were other explanations for color, and for the interference and diffraction effects that were observable at the time. Owing to Newton’s tremendous reputation, his view generally prevailed; the fact that Huygens’s principle worked was not considered direct evidence proving that light is a wave. The acceptance of the wave character of light came many years later in 1801, when the English physicist and physician Thomas Young (1773–1829) demonstrated optical interference with his now-classic double-slit experiment.

If there were not one but two sources of waves, the waves could be made to interfere, as in the case of waves on water (**Figure 3.2**). If light is an electromagnetic wave, it must therefore exhibit interference effects under appropriate circumstances. In Young's experiment, sunlight was passed through a pinhole on a board. The emerging beam fell on two pinholes on a second board. The light emanating from the two pinholes then fell on a screen where a pattern of bright and dark spots was observed. This pattern, called fringes, can only be explained through interference, a wave phenomenon.

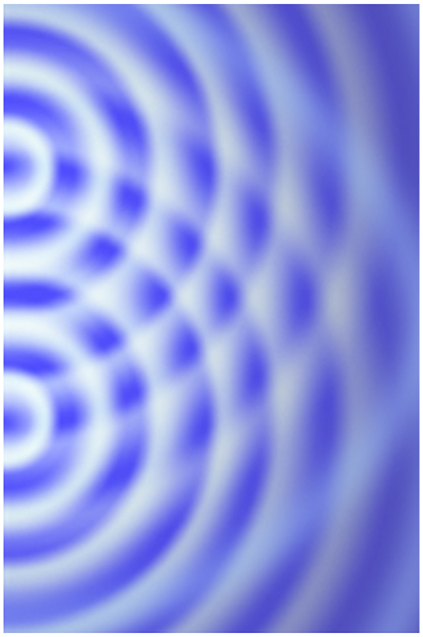


Figure 3.2 Photograph of an interference pattern produced by circular water waves in a ripple tank. Two thin plungers are vibrated up and down in phase at the surface of the water. Circular water waves are produced by and emanate from each plunger. The points where the water is calm (corresponding to destructive interference) are clearly visible.

We can analyze double-slit interference with the help of **Figure 3.3**, which depicts an apparatus analogous to Young's. Light from a monochromatic source falls on a slit S_0. The light emanating from S_0 is incident on two other slits S_1 and S_2 that are equidistant from S_0. A pattern of *interference fringes* on the screen is then produced by the light emanating from S_1 and S_2. All slits are assumed to be so narrow that they can be considered secondary point sources for Huygens' wavelets (**The Nature of Light**). Slits S_1 and S_2 are a distance *d* apart ($d \leq 1 \text{ mm}$), and the distance between the screen and the slits is $D(\approx 1 \text{ m})$, which is much greater than *d*.

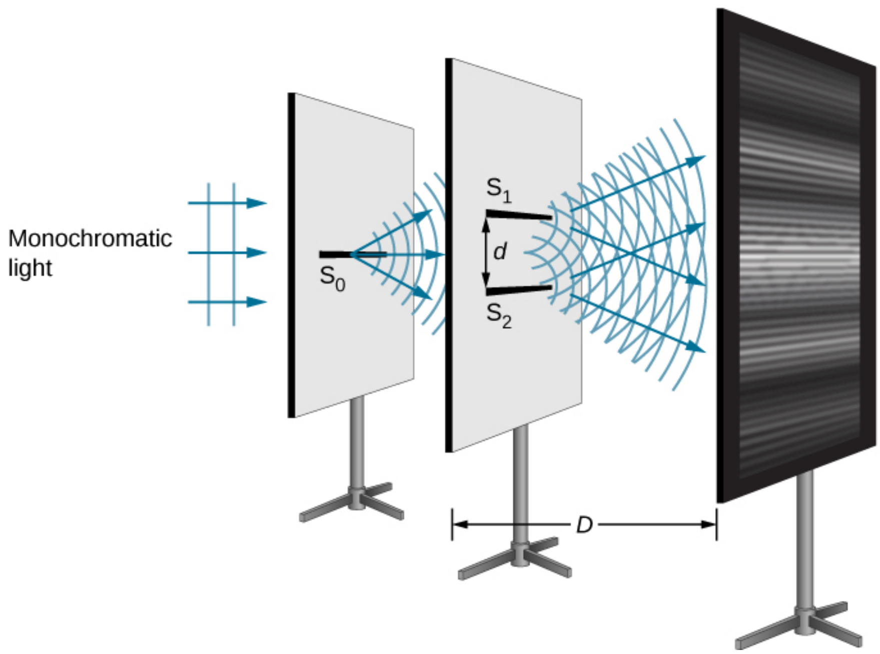


Figure 3.3 The double-slit interference experiment using monochromatic light and narrow slits. Fringes produced by interfering Huygens wavelets from slits S_1 and S_2 are observed on the screen.

Since S_0 is assumed to be a point source of monochromatic light, the secondary Huygens wavelets leaving S_1 and S_2 always maintain a constant phase difference (zero in this case because S_1 and S_2 are equidistant from S_0) and have the same frequency. The sources S_1 and S_2 are then said to be coherent. By **coherent waves**, we mean the waves are in phase or have a definite phase relationship. The term **incoherent** means the waves have random phase relationships, which would be the case if S_1 and S_2 were illuminated by two independent light sources, rather than a single source S_0. Two independent light sources (which may be two separate areas within the same lamp or the Sun) would generally not emit their light in unison, that is, not coherently. Also, because S_1 and S_2 are the same distance from S_0, the amplitudes of the two Huygens wavelets are equal.

Young used sunlight, where each wavelength forms its own pattern, making the effect more difficult to see. In the following discussion, we illustrate the double-slit experiment with **monochromatic** light (single λ) to clarify the effect. **Figure 3.4** shows the pure constructive and destructive interference of two waves having the same wavelength and amplitude.

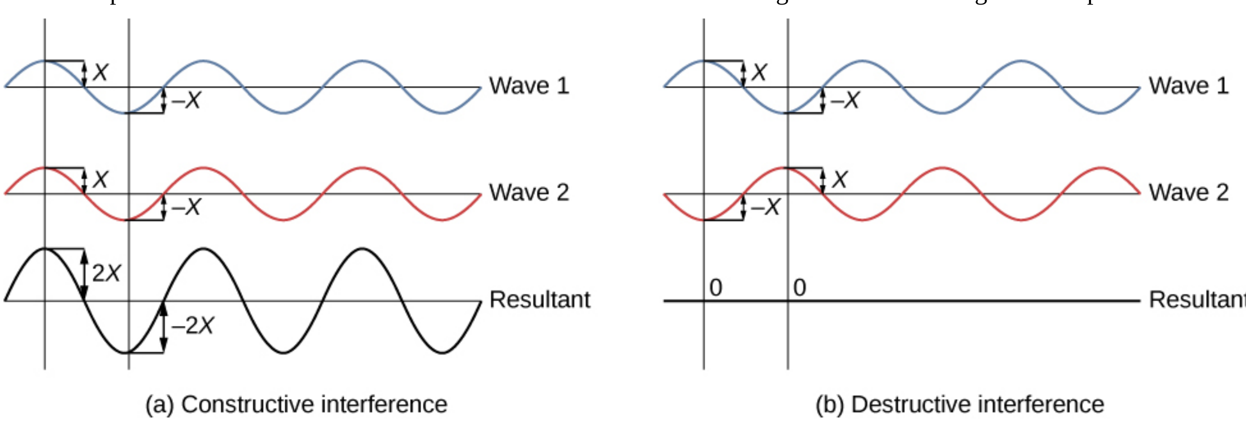


Figure 3.4 The amplitudes of waves add. (a) Pure constructive interference is obtained when identical waves are in phase. (b) Pure destructive interference occurs when identical waves are exactly out of phase, or shifted by half a wavelength.

When light passes through narrow slits, the slits act as sources of coherent waves and light spreads out as semicircular waves, as shown in **Figure 3.5**(a). Pure *constructive interference* occurs where the waves are crest to crest or trough to trough. Pure *destructive interference* occurs where they are crest to trough. The light must fall on a screen and be scattered into our eyes for us to see the pattern. An analogous pattern for water waves is shown in **Figure 3.2**. Note that regions of constructive and destructive interference move out from the slits at well-defined angles to the original beam. These angles depend on wavelength and the distance between the slits, as we shall see below.

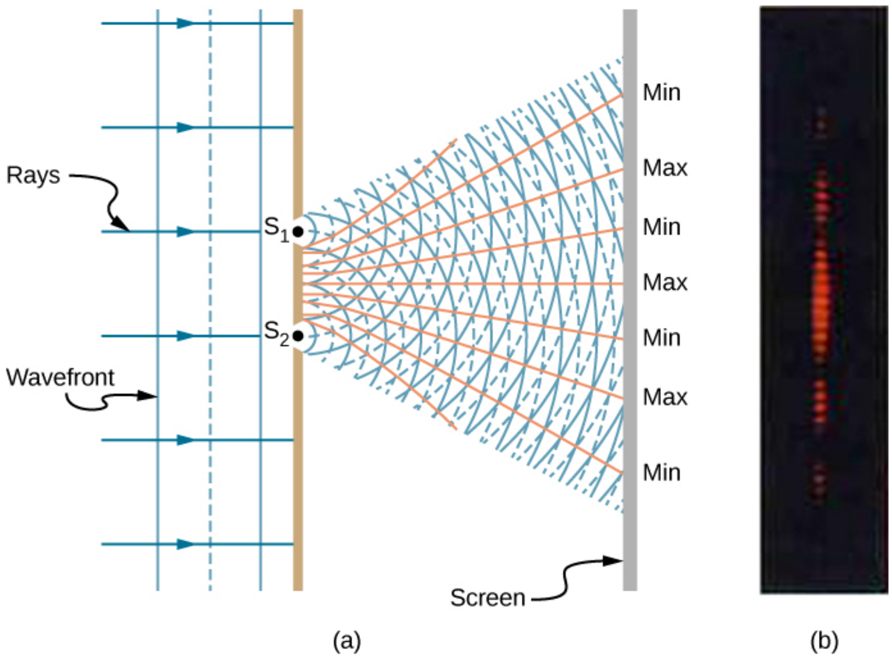


Figure 3.5 Double slits produce two coherent sources of waves that interfere. (a) Light spreads out (diffracts) from each slit, because the slits are narrow. These waves overlap and interfere constructively (bright lines) and destructively (dark regions). We can only see this if the light falls onto a screen and is scattered into our eyes. (b) When light that has passed through double slits falls on a screen, we see a pattern such as this.

To understand the double-slit interference pattern, consider how two waves travel from the slits to the screen (**Figure 3.6**). Each slit is a different distance from a given point on the screen. Thus, different numbers of wavelengths fit into each path. Waves start out from the slits in phase (crest to crest), but they may end up out of phase (crest to trough) at the screen if the paths differ in length by half a wavelength, interfering destructively. If the paths differ by a whole wavelength, then the waves arrive in phase (crest to crest) at the screen, interfering constructively. More generally, if the path length difference Δl between the two waves is any half-integral number of wavelengths [$(1 / 2)\lambda$, $(3 / 2)\lambda$, $(5 / 2)\lambda$, etc.], then destructive interference occurs. Similarly, if the path length difference is any integral number of wavelengths (λ, 2λ, 3λ, etc.), then constructive interference occurs. These conditions can be expressed as equations:

$$\Delta l = m\lambda, \quad \text{for } m = 0, \pm1, \pm2, \pm3 \ \ldots \ \text{(constructive interference)} \tag{3.1}$$

$$\Delta l = \left(m + \frac{1}{2}\right)\lambda, \quad \text{for } m = 0, \pm1, \pm2, \pm3 \ \ldots \ \text{(destructive interference)} \tag{3.2}$$

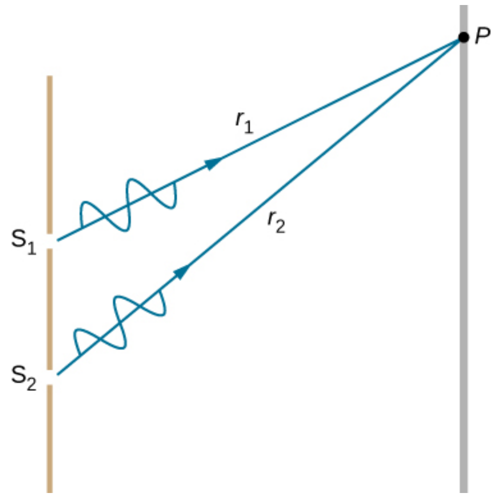


Figure 3.6 Waves follow different paths from the slits to a common point *P* on a screen. Destructive interference occurs where one path is a half wavelength longer than the other—the waves start in phase but arrive out of phase. Constructive interference occurs where one path is a whole wavelength longer than the other—the waves start out and arrive in phase.

3.2 | Mathematics of Interference

Learning Objectives

By the end of this section, you will be able to:

- Determine the angles for bright and dark fringes for double slit interference
- Calculate the positions of bright fringes on a screen

Figure 3.7(a) shows how to determine the path length difference Δl for waves traveling from two slits to a common point on a screen. If the screen is a large distance away compared with the distance between the slits, then the angle θ between the path and a line from the slits to the screen [part (b)] is nearly the same for each path. In other words, r_1 and r_2 are essentially parallel. The lengths of r_1 and r_2 differ by Δl, as indicated by the two dashed lines in the figure. Simple trigonometry shows

$$\Delta l = d \sin \theta \tag{3.3}$$

where *d* is the distance between the slits. Combining this result with **Equation 3.1**, we obtain constructive interference for a double slit when the path length difference is an integral multiple of the wavelength, or

$$d \sin \theta = m\lambda, \text{ for } m = 0, \pm 1, \pm 2, \pm 3, \ldots \text{ (constructive interference).} \tag{3.4}$$

Similarly, to obtain destructive interference for a double slit, the path length difference must be a half-integral multiple of the wavelength, or

$$d \sin \theta = (m + \frac{1}{2})\lambda, \text{ for } m = 0, \pm 1, \pm 2, \pm 3, \ldots \text{ (destructive interference)} \tag{3.5}$$

where λ is the wavelength of the light, *d* is the distance between slits, and θ is the angle from the original direction of the beam as discussed above. We call *m* the **order** of the interference. For example, $m = 4$ is fourth-order interference.

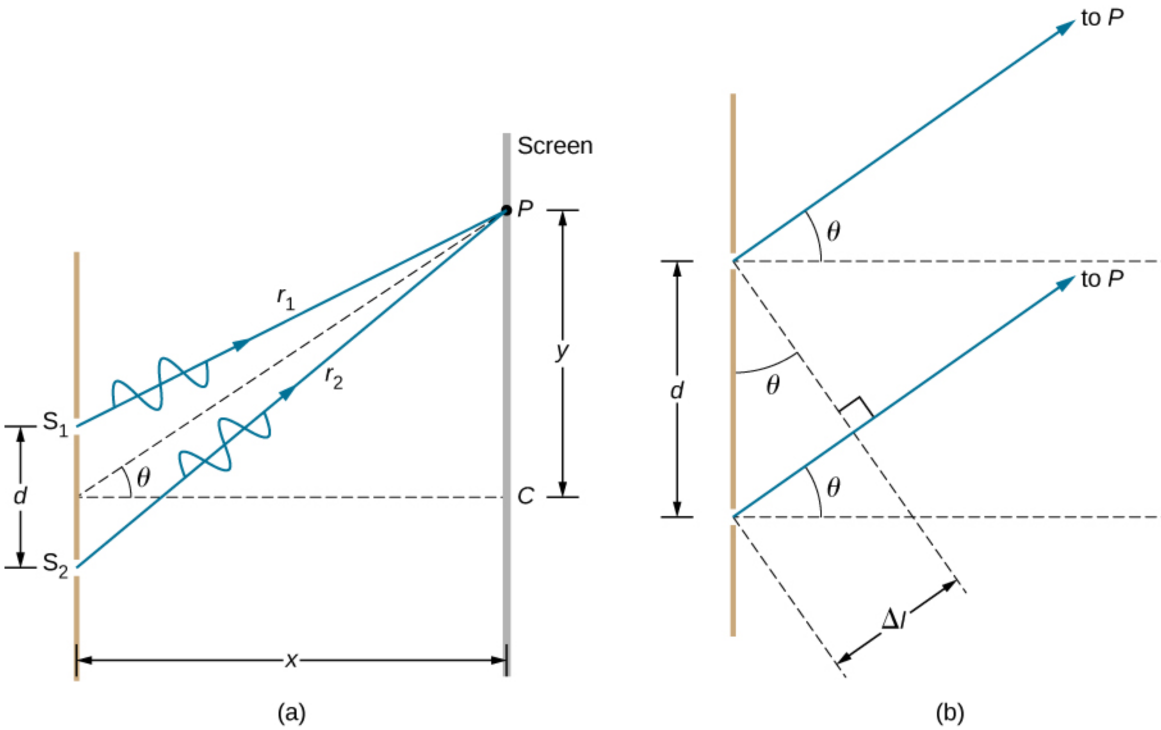


Figure 3.7 (a) To reach *P*, the light waves from S_1 and S_2 must travel different distances. (b) The path difference between the two rays is Δl.

The equations for double-slit interference imply that a series of bright and dark lines are formed. For vertical slits, the light spreads out horizontally on either side of the incident beam into a pattern called interference **fringes** (**Figure 3.8**). The closer the slits are, the more the bright fringes spread apart. We can see this by examining the equation

$d \sin\theta = m\lambda,$ for $m = 0, \pm 1, \pm 2, \pm 3\ldots$. For fixed λ and *m*, the smaller *d* is, the larger θ must be, since $\sin\theta = m\lambda/d$. This is consistent with our contention that wave effects are most noticeable when the object the wave encounters (here, slits a distance *d* apart) is small. Small *d* gives large θ, hence, a large effect.

Referring back to part (a) of the figure, θ is typically small enough that $\sin\theta \approx \tan\theta \approx y_m/D$, where y_m is the distance from the central maximum to the *m*th bright fringe and *D* is the distance between the slit and the screen. **Equation 3.4** may then be written as

$$d\frac{y_m}{D} = m\lambda$$

or

$$y_m = \frac{m\lambda D}{d}. \tag{3.6}$$

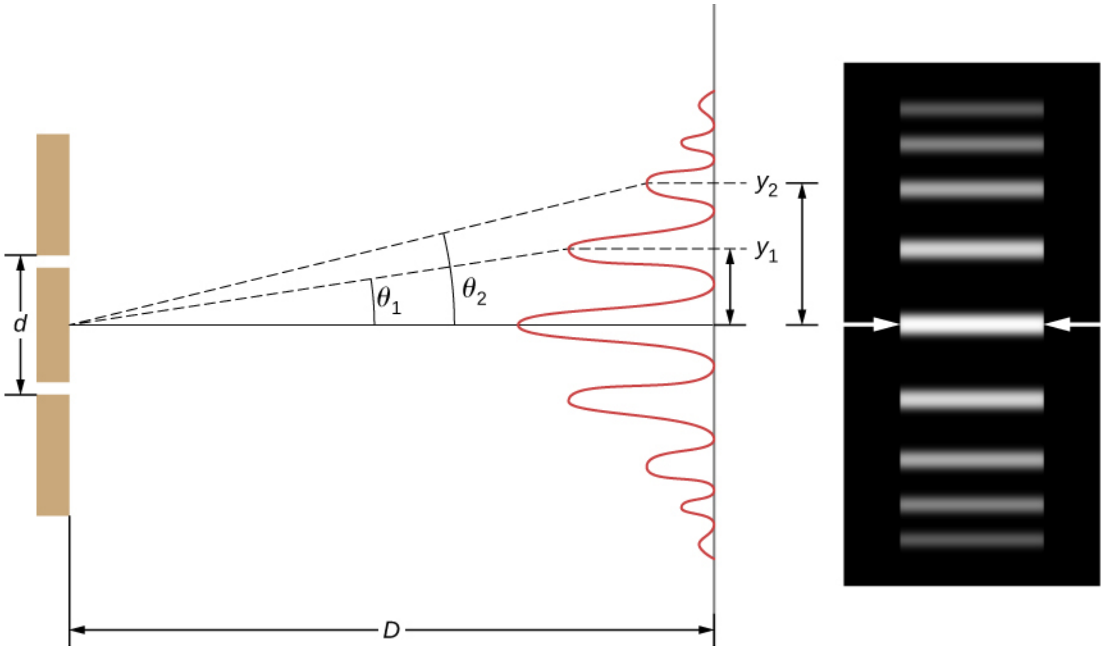


Figure 3.8 The interference pattern for a double slit has an intensity that falls off with angle. The image shows multiple bright and dark lines, or fringes, formed by light passing through a double slit.

Example 3.1

Finding a Wavelength from an Interference Pattern

Suppose you pass light from a He-Ne laser through two slits separated by 0.0100 mm and find that the third bright line on a screen is formed at an angle of $10.95°$ relative to the incident beam. What is the wavelength of the light?

Strategy

The phenomenon is two-slit interference as illustrated in **Figure 3.8** and the third bright line is due to third-order constructive interference, which means that $m = 3$. We are given $d = 0.0100\text{ mm}$ and $\theta = 10.95°$. The wavelength can thus be found using the equation $d \sin \theta = m\lambda$ for constructive interference.

Solution

Solving $d \sin \theta = m\lambda$ for the wavelength λ gives

$$\lambda = \frac{d \sin \theta}{m}.$$

Substituting known values yields

$$\lambda = \frac{(0.0100\text{ mm})(\sin 10.95°)}{3} = 6.33 \times 10^{-4}\text{ mm} = 633\text{ nm}.$$

Significance

To three digits, this is the wavelength of light emitted by the common He-Ne laser. Not by coincidence, this red color is similar to that emitted by neon lights. More important, however, is the fact that interference patterns can be used to measure wavelength. Young did this for visible wavelengths. This analytical techinque is still widely used to measure electromagnetic spectra. For a given order, the angle for constructive interference increases with λ , so that spectra (measurements of intensity versus wavelength) can be obtained.

Example 3.2

Calculating the Highest Order Possible

Interference patterns do not have an infinite number of lines, since there is a limit to how big *m* can be. What is the highest-order constructive interference possible with the system described in the preceding example?

Strategy

The equation $d \sin \theta = m\lambda$ (for $m = 0, \pm 1, \pm 2, \pm 3\ldots$) describes constructive interference from two slits. For fixed values of d and λ, the larger *m* is, the larger $\sin \theta$ is. However, the maximum value that $\sin \theta$ can have is 1, for an angle of $90°$. (Larger angles imply that light goes backward and does not reach the screen at all.) Let us find what value of *m* corresponds to this maximum diffraction angle.

Solution

Solving the equation $d \sin \theta = m\lambda$ for *m* gives

$$m = \frac{d \sin \theta}{\lambda}.$$

Taking $\sin \theta = 1$ and substituting the values of d and λ from the preceding example gives

$$m = \frac{(0.0100 \text{ mm})(1)}{633 \text{ nm}} \approx 15.8.$$

Therefore, the largest integer *m* can be is 15, or $m = 15$.

Significance

The number of fringes depends on the wavelength and slit separation. The number of fringes is very large for large slit separations. However, recall (see **The Propagation of Light** and the introduction for this chapter) that wave interference is only prominent when the wave interacts with objects that are not large compared to the wavelength. Therefore, if the slit separation and the sizes of the slits become much greater than the wavelength, the intensity pattern of light on the screen changes, so there are simply two bright lines cast by the slits, as expected, when light behaves like rays. We also note that the fringes get fainter farther away from the center. Consequently, not all 15 fringes may be observable.

3.1 Check Your Understanding In the system used in the preceding examples, at what angles are the first and the second bright fringes formed?

3.3 | Multiple-Slit Interference

Learning Objectives

By the end of this section, you will be able to:

- Describe the locations and intensities of secondary maxima for multiple-slit interference

Analyzing the interference of light passing through two slits lays out the theoretical framework of interference and gives us a historical insight into Thomas Young's experiments. However, much of the modern-day application of slit interference uses not just two slits but many, approaching infinity for practical purposes. The key optical element is called a diffraction grating, an important tool in optical analysis, which we discuss in detail in **Diffraction**. Here, we start the analysis of multiple-slit interference by taking the results from our analysis of the double slit ($N = 2$) and extending it to configurations with three, four, and much larger numbers of slits.

Figure 3.9 shows the simplest case of multiple-slit interference, with three slits, or $N = 3$. The spacing between slits is *d*, and the path length difference between adjacent slits is $d \sin \theta$, same as the case for the double slit. What is new is that the path length difference for the first and the third slits is $2d \sin \theta$. The condition for constructive interference is the same as for the double slit, that is

$$d \sin \theta = m\lambda.$$

When this condition is met, $2d \sin \theta$ is automatically a multiple of λ, so all three rays combine constructively, and the bright fringes that occur here are called **principal maxima**. But what happens when the path length difference between adjacent slits is only $\lambda/2$? We can think of the first and second rays as interfering destructively, but the third ray remains unaltered. Instead of obtaining a dark fringe, or a minimum, as we did for the double slit, we see a **secondary maximum** with intensity lower than the principal maxima.

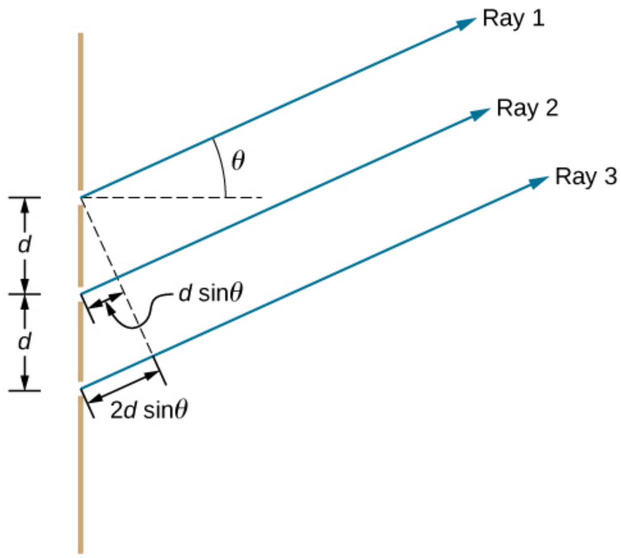


Figure 3.9 Interference with three slits. Different pairs of emerging rays can combine constructively or destructively at the same time, leading to secondary maxima.

In general, for N slits, these secondary maxima occur whenever an unpaired ray is present that does not go away due to destructive interference. This occurs at $(N - 2)$ evenly spaced positions between the principal maxima. The amplitude of the electromagnetic wave is correspondingly diminished to $1/N$ of the wave at the principal maxima, and the light intensity, being proportional to the square of the wave amplitude, is diminished to $1/N^2$ of the intensity compared to the principal maxima. As **Figure 3.10** shows, a dark fringe is located between every maximum (principal or secondary). As N grows larger and the number of bright and dark fringes increase, the widths of the maxima become narrower due to the closely located neighboring dark fringes. Because the total amount of light energy remains unaltered, narrower maxima require that each maximum reaches a correspondingly higher intensity.

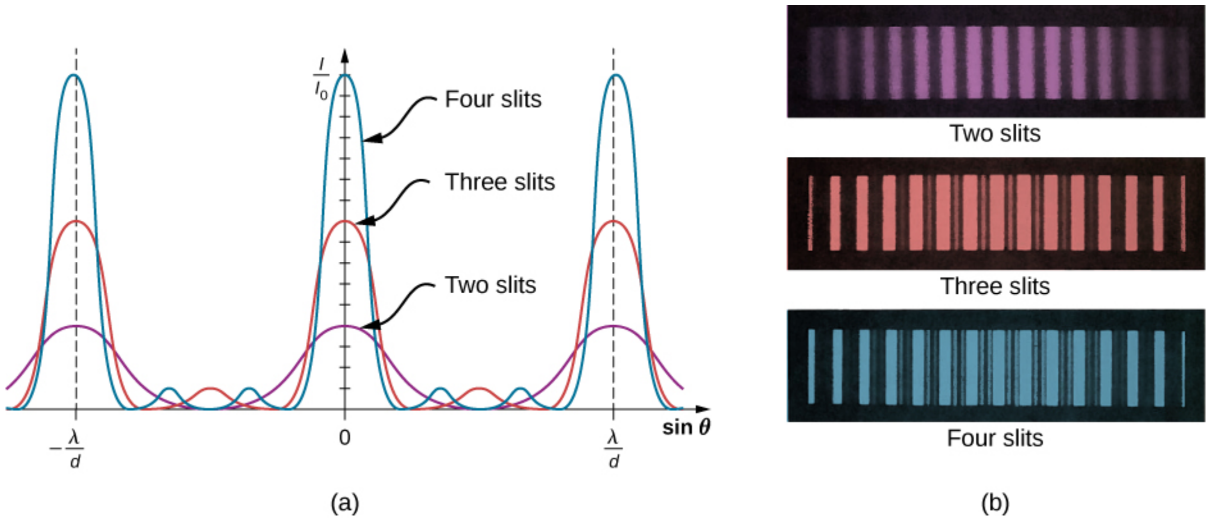


Figure 3.10 Interference fringe patterns for two, three and four slits. As the number of slits increases, more secondary maxima appear, but the principal maxima become brighter and narrower. (a) Graph and (b) photographs of fringe patterns.

3.4 | Interference in Thin Films

Learning Objectives

By the end of this section, you will be able to:

- Describe the phase changes that occur upon reflection
- Describe fringes established by reflected rays of a common source
- Explain the appearance of colors in thin films

The bright colors seen in an oil slick floating on water or in a sunlit soap bubble are caused by interference. The brightest colors are those that interfere constructively. This interference is between light reflected from different surfaces of a thin film; thus, the effect is known as **thin-film interference**.

As we noted before, interference effects are most prominent when light interacts with something having a size similar to its wavelength. A thin film is one having a thickness t smaller than a few times the wavelength of light, λ. Since color is associated indirectly with λ and because all interference depends in some way on the ratio of λ to the size of the object involved, we should expect to see different colors for different thicknesses of a film, as in **Figure 3.11**.

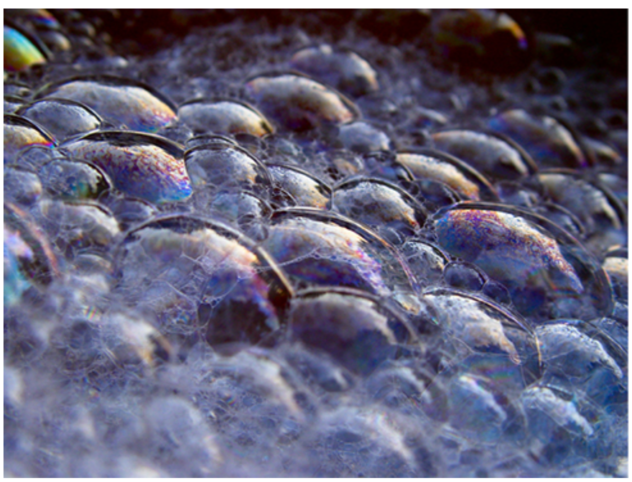


Figure 3.11 These soap bubbles exhibit brilliant colors when exposed to sunlight. (credit: Scott Robinson)

What causes thin-film interference? **Figure 3.12** shows how light reflected from the top and bottom surfaces of a film can interfere. Incident light is only partially reflected from the top surface of the film (ray 1). The remainder enters the film and is itself partially reflected from the bottom surface. Part of the light reflected from the bottom surface can emerge from the top of the film (ray 2) and interfere with light reflected from the top (ray 1). The ray that enters the film travels a greater distance, so it may be in or out of phase with the ray reflected from the top. However, consider for a moment, again, the bubbles in **Figure 3.11**. The bubbles are darkest where they are thinnest. Furthermore, if you observe a soap bubble carefully, you will note it gets dark at the point where it breaks. For very thin films, the difference in path lengths of rays 1 and 2 in **Figure 3.12** is negligible, so why should they interfere destructively and not constructively? The answer is that a phase change can occur upon reflection, as discussed next.

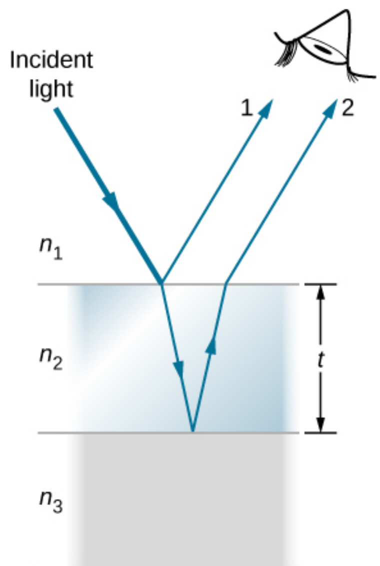


Figure 3.12 Light striking a thin film is partially reflected (ray 1) and partially refracted at the top surface. The refracted ray is partially reflected at the bottom surface and emerges as ray 2. These rays interfere in a way that depends on the thickness of the film and the indices of refraction of the various media.

Changes in Phase due to Reflection

We saw earlier (**Waves (http://cnx.org/content/m58367/latest/)**) that reflection of mechanical waves can involve a $180°$ phase change. For example, a traveling wave on a string is inverted (i.e., a $180°$ phase change) upon reflection at a boundary to which a heavier string is tied. However, if the second string is lighter (or more precisely, of a lower linear density), no inversion occurs. Light waves produce the same effect, but the deciding parameter for light is the index of refraction. Light waves undergo a $180°$ or π radians phase change upon reflection at an interface beyond which is a medium of higher index of refraction. No phase change takes place when reflecting from a medium of lower refractive index (**Figure 3.13**). Because of the periodic nature of waves, this phase change or inversion is equivalent to $\pm\lambda/2$ in distance travelled, or path length. Both the path length and refractive indices are important factors in thin-film interference.

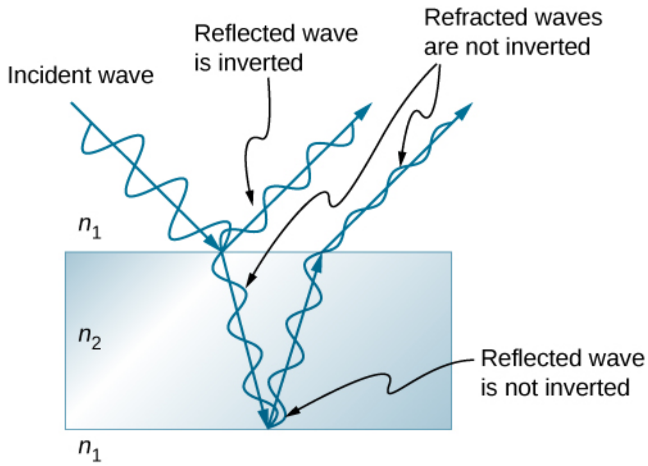


Figure 3.13 Reflection at an interface for light traveling from a medium with index of refraction n_1 to a medium with index of refraction n_2, $n_1 < n_2$, causes the phase of the wave to change by π radians.

If the film in **Figure 3.12** is a **soap bubble** (essentially water with air on both sides), then a phase shift of $\lambda/2$ occurs for ray 1 but not for ray 2. Thus, when the film is very thin and the path length difference between the two rays is negligible, they are exactly out of phase, and destructive interference occurs at all wavelengths. Thus, the soap bubble is dark here. The thickness of the film relative to the wavelength of light is the other crucial factor in thin-film interference. Ray 2 in **Figure 3.12** travels a greater distance than ray 1. For light incident perpendicular to the surface, ray 2 travels a distance approximately 2*t* farther than ray 1. When this distance is an integral or half-integral multiple of the wavelength in the medium $(\lambda_n = \lambda/n$, where λ is the wavelength in vacuum and *n* is the index of refraction), constructive or destructive interference occurs, depending also on whether there is a phase change in either ray.

Example 3.3

Calculating the Thickness of a Nonreflective Lens Coating

Sophisticated cameras use a series of several lenses. Light can reflect from the surfaces of these various lenses and degrade image clarity. To limit these reflections, lenses are coated with a thin layer of magnesium fluoride, which causes destructive thin-film interference. What is the thinnest this film can be, if its index of refraction is 1.38 and it is designed to limit the reflection of 550-nm light, normally the most intense visible wavelength? Assume the index of refraction of the glass is 1.52.

Strategy

Refer to **Figure 3.12** and use $n_1 = 1.00$ for air, $n_2 = 1.38$, and $n_3 = 1.52$. Both ray 1 and ray 2 have a $\lambda/2$ shift upon reflection. Thus, to obtain destructive interference, ray 2 needs to travel a half wavelength farther than ray 1. For rays incident perpendicularly, the path length difference is 2*t*.

Solution

To obtain destructive interference here,

$$2t = \frac{\lambda_{n2}}{2}$$

where λ_{n2} is the wavelength in the film and is given by $\lambda_{n2} = \lambda/n_2$. Thus,

$$2t = \frac{\lambda/n_2}{2}.$$

Solving for *t* and entering known values yields

$$t = \frac{\lambda/n_2}{4} = \frac{(500\text{ nm})/1.38}{4} = 90.6\text{ nm}.$$

Significance

Films such as the one in this example are most effective in producing destructive interference when the thinnest layer is used, since light over a broader range of incident angles is reduced in intensity. These films are called nonreflective coatings; this is only an approximately correct description, though, since other wavelengths are only partially cancelled. Nonreflective coatings are also used in car windows and sunglasses.

Combining Path Length Difference with Phase Change

Thin-film interference is most constructive or most destructive when the path length difference for the two rays is an integral or half-integral wavelength. That is, for rays incident perpendicularly,

$$2t = \lambda_n, 2\lambda_n, 3\lambda_n, \ldots \text{ or } 2t = \lambda_n/2, 3\lambda_n/2, 5\lambda_n/2, \ldots.$$

To know whether interference is constructive or destructive, you must also determine if there is a phase change upon reflection. Thin-film interference thus depends on film thickness, the wavelength of light, and the refractive indices. For white light incident on a film that varies in thickness, you can observe rainbow colors of constructive interference for various wavelengths as the thickness varies.

Example 3.4

Soap Bubbles

(a) What are the three smallest thicknesses of a soap bubble that produce constructive interference for red light with a wavelength of 650 nm? The index of refraction of soap is taken to be the same as that of water. (b) What three smallest thicknesses give destructive interference?

Strategy

Use **Figure 3.12** to visualize the bubble, which acts as a thin film between two layers of air. Thus $n_1 = n_3 = 1.00$ for air, and $n_2 = 1.333$ for soap (equivalent to water). There is a $\lambda/2$ shift for ray 1 reflected from the top surface of the bubble and no shift for ray 2 reflected from the bottom surface. To get constructive interference, then, the path length difference ($2t$) must be a half-integral multiple of the wavelength—the first three being $\lambda_n/2, 3\lambda_n/2$, and $5\lambda_n/2$. To get destructive interference, the path length difference must be an integral multiple of the wavelength—the first three being 0, λ_n, and $2\lambda_n$.

Solution

a. Constructive interference occurs here when

$$2t_c = \frac{\lambda_n}{2}, \frac{3\lambda_n}{2}, \frac{5\lambda_n}{2}, \ \ldots .$$

Thus, the smallest constructive thickness t_c is

$$t_c = \frac{\lambda_n}{4} = \frac{\lambda/n}{4} = \frac{(650\text{ nm})/1.333}{4} = 122\text{ nm}.$$

The next thickness that gives constructive interference is $t'_c = 3\lambda_n/4$, so that

$$t'_c = 366\text{ nm}.$$

Finally, the third thickness producing constructive interference is $t'_c = 5\lambda_n/4$, so that

$$t'_c = 610\text{ nm}.$$

b. For destructive interference, the path length difference here is an integral multiple of the wavelength. The first occurs for zero thickness, since there is a phase change at the top surface, that is,

$$t_{\mathrm{d}} = 0,$$

the very thin (or negligibly thin) case discussed above. The first non-zero thickness producing destructive interference is

$$2t'_{\mathrm{d}} = \lambda_n.$$

Substituting known values gives

$$t'_{\mathrm{d}} = \frac{\lambda}{2} = \frac{\lambda/n}{2} = \frac{(650\ \mathrm{nm})/1.333}{2} = 244\ \mathrm{nm}.$$

Finally, the third destructive thickness is $2t''_{\mathrm{d}} = 2\lambda_n$, so that

$$t''_{\mathrm{d}} = \lambda_n = \frac{\lambda}{n} = \frac{650\ \mathrm{nm}}{1.333} = 488\ \mathrm{nm}.$$

Significance

If the bubble were illuminated with pure red light, we would see bright and dark bands at very uniform increases in thickness. First would be a dark band at 0 thickness, then bright at 122 nm thickness, then dark at 244 nm, bright at 366 nm, dark at 488 nm, and bright at 610 nm. If the bubble varied smoothly in thickness, like a smooth wedge, then the bands would be evenly spaced.

3.2 Check Your Understanding Going further with **Example 3.4**, what are the next two thicknesses of soap bubble that would lead to (a) constructive interference, and (b) destructive interference?

Another example of thin-film interference can be seen when microscope slides are separated (see **Figure 3.14**). The slides are very flat, so that the wedge of air between them increases in thickness very uniformly. A phase change occurs at the second surface but not the first, so a dark band forms where the slides touch. The rainbow colors of constructive interference repeat, going from violet to red again and again as the distance between the slides increases. As the layer of air increases, the bands become more difficult to see, because slight changes in incident angle have greater effects on path length differences. If monochromatic light instead of white light is used, then bright and dark bands are obtained rather than repeating rainbow colors.

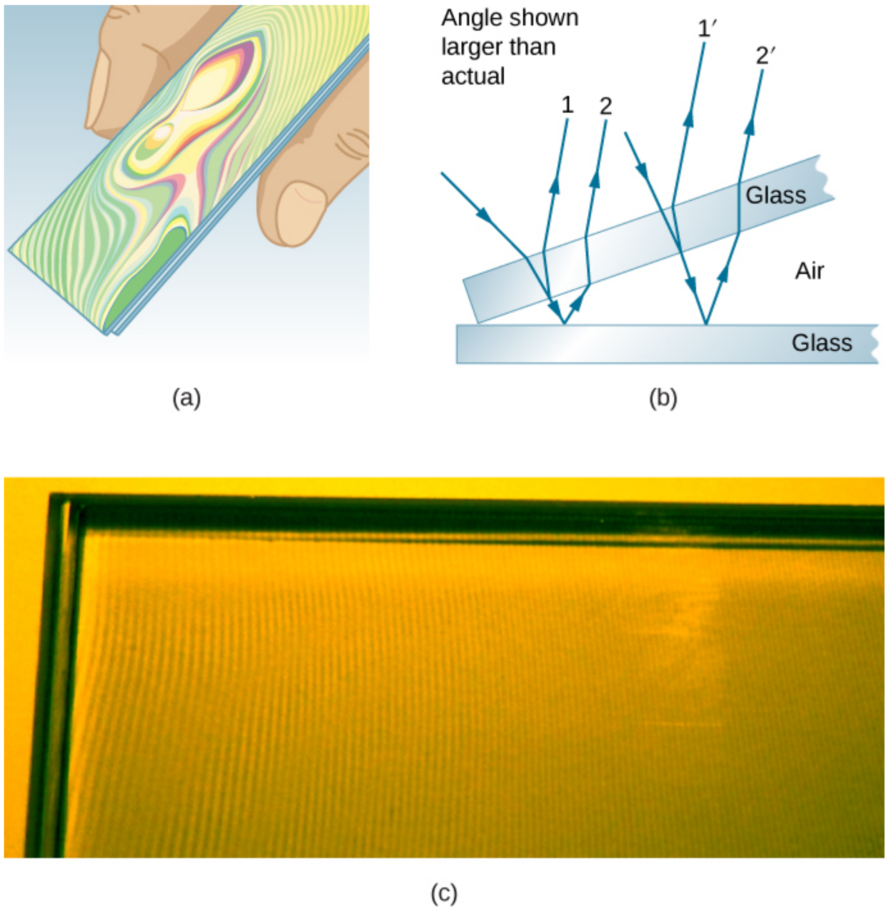


Figure 3.14 (a) The rainbow-color bands are produced by thin-film interference in the air between the two glass slides. (b) Schematic of the paths taken by rays in the wedge of air between the slides. (c) If the air wedge is illuminated with monochromatic light, bright and dark bands are obtained rather than repeating rainbow colors.

An important application of thin-film interference is found in the manufacturing of optical instruments. A lens or mirror can be compared with a master as it is being ground, allowing it to be shaped to an accuracy of less than a wavelength over its entire surface. **Figure 3.15** illustrates the phenomenon called **Newton's rings**, which occurs when the plane surfaces of two lenses are placed together. (The circular bands are called Newton's rings because Isaac Newton described them and their use in detail. Newton did not discover them; Robert Hooke did, and Newton did not believe they were due to the wave character of light.) Each successive ring of a given color indicates an increase of only half a wavelength in the distance between the lens and the blank, so that great precision can be obtained. Once the lens is perfect, no rings appear.

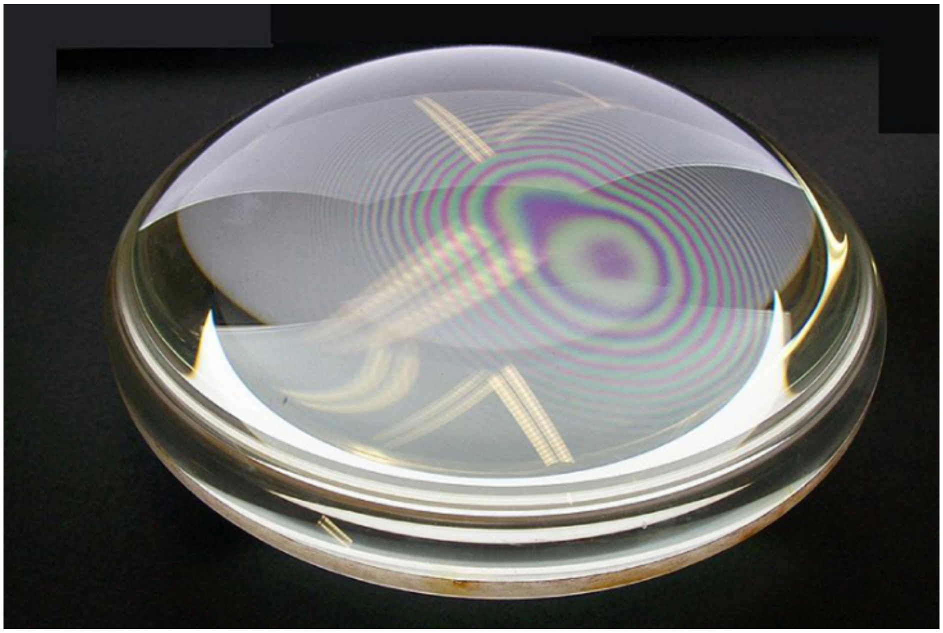


Figure 3.15 “Newton’s rings” interference fringes are produced when two plano-convex lenses are placed together with their plane surfaces in contact. The rings are created by interference between the light reflected off the two surfaces as a result of a slight gap between them, indicating that these surfaces are not precisely plane but are slightly convex. (credit: Ulf Seifert)

Thin-film interference has many other applications, both in nature and in manufacturing. The wings of certain moths and butterflies have nearly iridescent colors due to thin-film interference. In addition to pigmentation, the wing’s color is affected greatly by constructive interference of certain wavelengths reflected from its film-coated surface. Some car manufacturers offer special paint jobs that use thin-film interference to produce colors that change with angle. This expensive option is based on variation of thin-film path length differences with angle. Security features on credit cards, banknotes, driving licenses, and similar items prone to forgery use thin-film interference, diffraction gratings, or holograms. As early as 1998, Australia led the way with dollar bills printed on polymer with a diffraction grating security feature, making the currency difficult to forge. Other countries, such as Canada, New Zealand, and Taiwan, are using similar technologies, while US currency includes a thin-film interference effect.

3.5 | The Michelson Interferometer

Learning Objectives

By the end of this section, you will be able to:

- Explain changes in fringes observed with a Michelson interferometer caused by mirror movements
- Explain changes in fringes observed with a Michelson interferometer caused by changes in medium

The Michelson **interferometer** (invented by the American physicist Albert A. Michelson, 1852–1931) is a precision instrument that produces interference fringes by splitting a light beam into two parts and then recombining them after they have traveled different optical paths. **Figure 3.16** depicts the interferometer and the path of a light beam from a single point on the extended source S, which is a ground-glass plate that diffuses the light from a monochromatic lamp of wavelength λ_0. The beam strikes the half-silvered mirror M, where half of it is reflected to the side and half passes through the mirror. The reflected light travels to the movable plane mirror M_1, where it is reflected back through M to the observer. The transmitted half of the original beam is reflected back by the stationary mirror M_2 and then toward the observer by M.

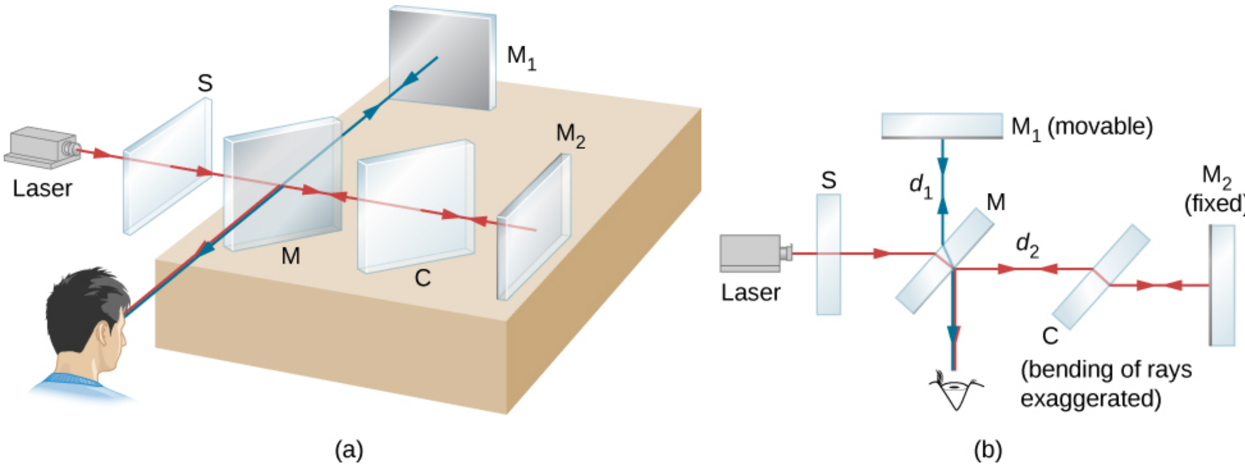


Figure 3.16 (a) The Michelson interferometer. The extended light source is a ground-glass plate that diffuses the light from a laser. (b) A planar view of the interferometer.

Because both beams originate from the same point on the source, they are coherent and therefore interfere. Notice from the figure that one beam passes through M three times and the other only once. To ensure that both beams traverse the same thickness of glass, a compensator plate C of transparent glass is placed in the arm containing M_2. This plate is a duplicate of M (without the silvering) and is usually cut from the same piece of glass used to produce M. With the compensator in place, any phase difference between the two beams is due solely to the difference in the distances they travel.

The path difference of the two beams when they recombine is $2d_1 - 2d_2$, where d_1 is the distance between M and M_1, and d_2 is the distance between M and M_2. Suppose this path difference is an integer number of wavelengths $m\lambda_0$. Then, constructive interference occurs and a bright image of the point on the source is seen at the observer. Now the light from any other point on the source whose two beams have this same path difference also undergoes constructive interference and produces a bright image. The collection of these point images is a bright fringe corresponding to a path difference of $m\lambda_0$ (**Figure 3.17**). When M_1 is moved a distance $\Delta d = \lambda_0/2$, this path difference changes by λ_0, and each fringe moves to the position previously occupied by an adjacent fringe. Consequently, by counting the number of fringes *m* passing a given point as M_1 is moved, an observer can measure minute displacements that are accurate to a fraction of a wavelength, as shown by the relation

$$\Delta d = m\frac{\lambda_0}{2}. \quad (3.7)$$

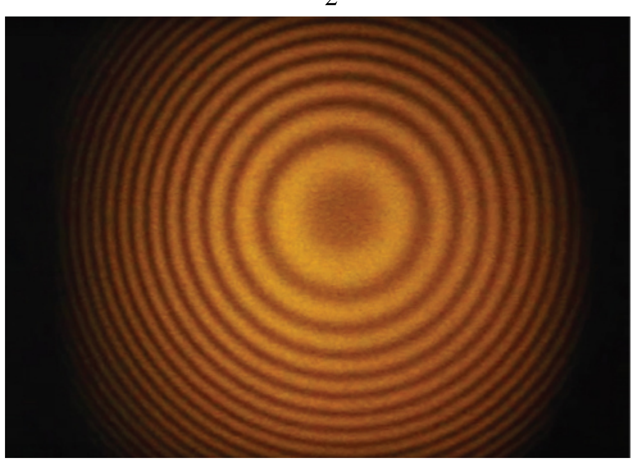


Figure 3.17 Fringes produced with a Michelson interferometer. (credit: “SILLAGESvideos”/YouTube)

Example 3.5

Precise Distance Measurements by Michelson Interferometer

A red laser light of wavelength 630 nm is used in a Michelson interferometer. While keeping the mirror M_1 fixed, mirror M_2 is moved. The fringes are found to move past a fixed cross-hair in the viewer. Find the distance the mirror M_2 is moved for a single fringe to move past the reference line.

Strategy

Refer to **Figure 3.16** for the geometry. We use the result of the Michelson interferometer interference condition to find the distance moved, Δd.

Solution

For a 630-nm red laser light, and for each fringe crossing $(m = 1)$, the distance traveled by M_2 if you keep M_1 fixed is

$$\Delta d = m\frac{\lambda_0}{2} = 1 \times \frac{630\text{ nm}}{2} = 315\text{ nm} = 0.315\,\mu\text{m}.$$

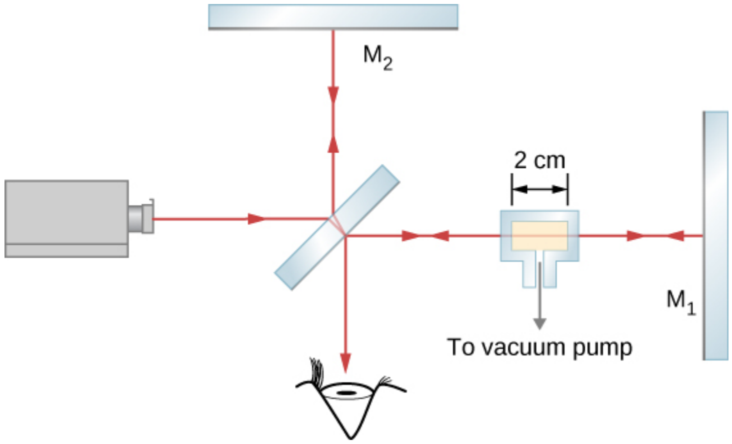
Significance

An important application of this measurement is the definition of the standard meter. As mentioned in **Units and Measurement (http://cnx.org/content/m58268/latest/)**, the length of the standard meter was once defined as the mirror displacement in a Michelson interferometer corresponding to 1,650,763.73 wavelengths of the particular fringe of krypton-86 in a gas discharge tube.

Example 3.6

Measuring the Refractive Index of a Gas

In one arm of a Michelson interferometer, a glass chamber is placed with attachments for evacuating the inside and putting gases in it. The space inside the container is 2 cm wide. Initially, the container is empty. As gas is slowly let into the chamber, you observe that dark fringes move past a reference line in the field of observation. By the time the chamber is filled to the desired pressure, you have counted 122 fringes move past the reference line. The wavelength of the light used is 632.8 nm. What is the refractive index of this gas?



Strategy

The $m = 122$ fringes observed compose the difference between the number of wavelengths that fit within the empty chamber (vacuum) and the number of wavelengths that fit within the same chamber when it is gas-filled. The wavelength in the filled chamber is shorter by a factor of n, the index of refraction.

Solution

The ray travels a distance $t = 2\text{ cm}$ to the right through the glass chamber and another distance t to the left upon reflection. The total travel is $L = 2t$. When empty, the number of wavelengths that fit in this chamber is

$$N_0 = \frac{L}{\lambda_0} = \frac{2t}{\lambda_0}$$

where $\lambda_0 = 632.8\text{ nm}$ is the wavelength in vacuum of the light used. In any other medium, the wavelength is $\lambda = \lambda_0/n$ and the number of wavelengths that fit in the gas-filled chamber is

$$N = \frac{L}{\lambda} = \frac{2t}{\lambda_0/n}.$$

The number of fringes observed in the transition is

$$\begin{aligned} m &= N - N_0, \\ &= \frac{2t}{\lambda_0/n} - \frac{2t}{\lambda_0}, \\ &= \frac{2t}{\lambda_0}(n-1). \end{aligned}$$

Solving for $(n - 1)$ gives

$$n - 1 = m\left(\frac{\lambda_0}{2t}\right) = 122\left(\frac{632.8 \times 10^{-9}\text{ m}}{2(2 \times 10^{-2}\text{ m})}\right) = 0.0019$$

and $n = 1.0019$.

Significance

The indices of refraction for gases are so close to that of vacuum, that we normally consider them equal to 1. The difference between 1 and 1.0019 is so small that measuring it requires a correspondingly sensitive technique such as interferometry. We cannot, for example, hope to measure this value using techniques based simply on Snell's law.

3.3 Check Your Understanding Although m, the number of fringes observed, is an integer, which is often regarded as having zero uncertainty, in practical terms, it is all too easy to lose track when counting fringes. In **Example 3.6**, if you estimate that you might have missed as many as five fringes when you reported $m = 122$ fringes, (a) is the value for the index of refraction worked out in **Example 3.6** too large or too small? (b) By how much?

Problem-Solving Strategy: Wave Optics

Step 1. *Examine the situation to determine that interference is involved*. Identify whether slits, thin films, or interferometers are considered in the problem.

Step 2. *If slits are involved*, note that diffraction gratings and double slits produce very similar interference patterns, but that gratings have narrower (sharper) maxima. Single-slit patterns are characterized by a large central maximum and smaller maxima to the sides.

Step 3. *If thin-film interference or an interferometer is involved, take note of the path length difference between the two rays that interfere*. Be certain to use the wavelength in the medium involved, since it differs from the wavelength in vacuum. Note also that there is an additional $\lambda/2$ phase shift when light reflects from a medium with a greater index of refraction.

Step 4. *Identify exactly what needs to be determined in the problem (identify the unknowns)*. A written list is useful. Draw a diagram of the situation. Labeling the diagram is useful.

Step 5. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns)*.

Step 6. *Solve the appropriate equation for the quantity to be determined (the unknown) and enter the knowns*. Slits, gratings, and the Rayleigh limit involve equations.

Step 7. *For thin-film interference, you have constructive interference for a total shift that is an integral number of wavelengths. You have destructive interference for a total shift of a half-integral number of wavelengths*. Always keep in mind that crest to crest is constructive whereas crest to trough is destructive.

Step 8. *Check to see if the answer is reasonable: Does it make sense?* Angles in interference patterns cannot be greater than $90°$, for example.

CHAPTER 3 REVIEW

KEY TERMS

coherent waves waves are in phase or have a definite phase relationship

fringes bright and dark patterns of interference

incoherent waves have random phase relationships

interferometer instrument that uses interference of waves to make measurements

monochromatic light composed of one wavelength only

Newton's rings circular interference pattern created by interference between the light reflected off two surfaces as a result of a slight gap between them

order integer m used in the equations for constructive and destructive interference for a double slit

principal maximum brightest interference fringes seen with multiple slits

secondary maximum bright interference fringes of intensity lower than the principal maxima

thin-film interference interference between light reflected from different surfaces of a thin film

KEY EQUATIONS

Constructive interference	$\Delta l = m\lambda,$ for m = 0, ±1, ±2, ±3…
Destructive interference	$\Delta l = (m + \frac{1}{2})\lambda,$ for m = 0, ±1, ±2, ±3…
Path length difference for waves from two slits to a common point on a screen	$\Delta l = d \sin \theta$
Constructive interference	$d \sin \theta = m\lambda, \quad \text{for } m = 0, \pm1, \pm2, \pm3, \ldots$
Destructive interference	$d \sin \theta = (m + \frac{1}{2})\lambda, \quad \text{for } m = 0, \pm1, \pm2, \pm3, \ldots$
Distance from central maximum to the mth bright fringe	$y_m = \frac{m\lambda D}{d}$
Displacement measured by a Michelson interferometer	$\Delta d = m\frac{\lambda_0}{2}$

SUMMARY

3.1 Young's Double-Slit Interference

- Young's double-slit experiment gave definitive proof of the wave character of light.
- An interference pattern is obtained by the superposition of light from two slits.

3.2 Mathematics of Interference

- In double-slit diffraction, constructive interference occurs when $d \sin \theta = m\lambda (\text{for } m = 0, \pm1, \pm2, \pm3 \ldots)$, where d is the distance between the slits, θ is the angle relative to the incident direction, and m is the order of the interference.
- Destructive interference occurs when $d \sin \theta = (m + \frac{1}{2})\lambda$ for $m = 0, \pm1, \pm2, \pm3, \ldots$.

3.3 Multiple-Slit Interference

- Interference from multiple slits ($N > 2$) produces principal as well as secondary maxima.
- As the number of slits is increased, the intensity of the principal maxima increases and the width decreases.

3.4 Interference in Thin Films

- When light reflects from a medium having an index of refraction greater than that of the medium in which it is traveling, a $180°$ phase change (or a $\lambda/2$ shift) occurs.
- Thin-film interference occurs between the light reflected from the top and bottom surfaces of a film. In addition to the path length difference, there can be a phase change.

3.5 The Michelson Interferometer

- When the mirror in one arm of the interferometer moves a distance of $\lambda/2$ each fringe in the interference pattern moves to the position previously occupied by the adjacent fringe.

CONCEPTUAL QUESTIONS

3.1 Young's Double-Slit Interference

1. Young's double-slit experiment breaks a single light beam into two sources. Would the same pattern be obtained for two independent sources of light, such as the headlights of a distant car? Explain.

2. Is it possible to create a experimental setup in which there is only destructive interference? Explain.

3. Why won't two small sodium lamps, held close together, produce an interference pattern on a distant screen? What if the sodium lamps were replaced by two laser pointers held close together?

3.2 Mathematics of Interference

4. Suppose you use the same double slit to perform Young's double-slit experiment in air and then repeat the experiment in water. Do the angles to the same parts of the interference pattern get larger or smaller? Does the color of the light change? Explain.

5. Why is monochromatic light used in the double slit experiment? What would happen if white light were used?

3.4 Interference in Thin Films

6. What effect does increasing the wedge angle have on the spacing of interference fringes? If the wedge angle is too large, fringes are not observed. Why?

7. How is the difference in paths taken by two originally in-phase light waves related to whether they interfere constructively or destructively? How can this be affected by reflection? By refraction?

8. Is there a phase change in the light reflected from either surface of a contact lens floating on a person's tear layer? The index of refraction of the lens is about 1.5, and its top surface is dry.

9. In placing a sample on a microscope slide, a glass cover is placed over a water drop on the glass slide. Light incident from above can reflect from the top and bottom of the glass cover and from the glass slide below the water drop. At which surfaces will there be a phase change in the reflected light?

10. Answer the above question if the fluid between the two pieces of crown glass is carbon disulfide.

11. While contemplating the food value of a slice of ham, you notice a rainbow of color reflected from its moist surface. Explain its origin.

12. An inventor notices that a soap bubble is dark at its thinnest and realizes that destructive interference is taking place for all wavelengths. How could she use this knowledge to make a nonreflective coating for lenses that is effective at all wavelengths? That is, what limits would there be on the index of refraction and thickness of the coating? How might this be impractical?

13. A nonreflective coating like the one described in **Example 3.3** works ideally for a single wavelength and for perpendicular incidence. What happens for other wavelengths and other incident directions? Be specific.

14. Why is it much more difficult to see interference fringes for light reflected from a thick piece of glass than from a thin film? Would it be easier if monochromatic light were used?

3.5 The Michelson Interferometer

15. Describe how a Michelson interferometer can be used to measure the index of refraction of a gas (including air).

PROBLEMS

3.2 Mathematics of Interference

16. At what angle is the first-order maximum for 450-nm wavelength blue light falling on double slits separated by 0.0500 mm?

17. Calculate the angle for the third-order maximum of 580-nm wavelength yellow light falling on double slits separated by 0.100 mm.

18. What is the separation between two slits for which 610-nm orange light has its first maximum at an angle of $30.0°$?

19. Find the distance between two slits that produces the first minimum for 410-nm violet light at an angle of $45.0°$.

20. Calculate the wavelength of light that has its third minimum at an angle of $30.0°$ when falling on double slits separated by $3.00\ \mu\text{m}$. Explicitly show how you follow the steps from the **Problem-Solving Strategy: Wave Optics**, located at the end of the chapter.

21. What is the wavelength of light falling on double slits separated by $2.00\ \mu\text{m}$ if the third-order maximum is at an angle of $60.0°$?

22. At what angle is the fourth-order maximum for the situation in the preceding problem?

23. What is the highest-order maximum for 400-nm light falling on double slits separated by $25.0\ \mu\text{m}$?

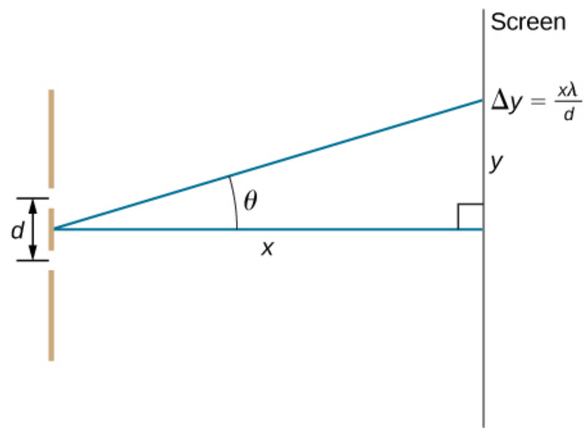
24. Find the largest wavelength of light falling on double slits separated by $1.20\ \mu\text{m}$ for which there is a first-order maximum. Is this in the visible part of the spectrum?

25. What is the smallest separation between two slits that will produce a second-order maximum for 720-nm red light?

26. (a) What is the smallest separation between two slits that will produce a second-order maximum for any visible light? (b) For all visible light?

27. (a) If the first-order maximum for monochromatic light falling on a double slit is at an angle of $10.0°$, at what angle is the second-order maximum? (b) What is the angle of the first minimum? (c) What is the highest-order maximum possible here?

28. Shown below is a double slit located a distance *x* from a screen, with the distance from the center of the screen given by *y*. When the distance *d* between the slits is relatively large, numerous bright spots appear, called fringes. Show that, for small angles (where $\sin\theta \approx \theta$, with θ in radians), the distance between fringes is given by $\Delta y = x\lambda/d$



29. Using the result of the preceding problem, (a) calculate the distance between fringes for 633-nm light falling on double slits separated by 0.0800 mm, located 3.00 m from a screen. (b) What would be the distance between fringes if the entire apparatus were submersed in water, whose index of refraction is 1.33?

30. Using the result of the problem two problems prior, find the wavelength of light that produces fringes 7.50 mm apart on a screen 2.00 m from double slits separated by 0.120 mm.

31. In a double-slit experiment, the fifth maximum is 2.8 cm from the central maximum on a screen that is 1.5 m away from the slits. If the slits are 0.15 mm apart, what is the wavelength of the light being used?

32. The source in Young's experiment emits at two wavelengths. On the viewing screen, the fourth maximum for one wavelength is located at the same spot as the fifth maximum for the other wavelength. What is the ratio of the two wavelengths?

33. If 500-nm and 650-nm light illuminates two slits that are separated by 0.50 mm, how far apart are the second-order maxima for these two wavelengths on a screen 2.0 m away?

34. Red light of wavelength of 700 nm falls on a double slit separated by 400 nm. (a) At what angle is the first-order maximum in the diffraction pattern? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

3.3 Multiple-Slit Interference

35. Ten narrow slits are equally spaced 0.25 mm apart and illuminated with yellow light of wavelength 580 nm. (a) What are the angular positions of the third and fourth principal maxima? (b) What is the separation of these maxima on a screen 2.0 m from the slits?

36. The width of bright fringes can be calculated as the separation between the two adjacent dark fringes on either side. Find the angular widths of the third- and fourth-order bright fringes from the preceding problem.

37. For a three-slit interference pattern, find the ratio of the peak intensities of a secondary maximum to a principal maximum.

38. What is the angular width of the central fringe of the interference pattern of (a) 20 slits separated by $d = 2.0 \times 10^{-3}\,\text{mm}$? (b) 50 slits with the same separation? Assume that $\lambda = 600\,\text{nm}$.

3.4 Interference in Thin Films

39. A soap bubble is 100 nm thick and illuminated by white light incident perpendicular to its surface. What wavelength and color of visible light is most constructively reflected, assuming the same index of refraction as water?

40. An oil slick on water is 120 nm thick and illuminated by white light incident perpendicular to its surface. What color does the oil appear (what is the most constructively reflected wavelength), given its index of refraction is 1.40?

41. Calculate the minimum thickness of an oil slick on water that appears blue when illuminated by white light perpendicular to its surface. Take the blue wavelength to be 470 nm and the index of refraction of oil to be 1.40.

42. Find the minimum thickness of a soap bubble that appears red when illuminated by white light perpendicular to its surface. Take the wavelength to be 680 nm, and assume the same index of refraction as water.

43. A film of soapy water ($n = 1.33$) on top of a plastic cutting board has a thickness of 233 nm. What color is most strongly reflected if it is illuminated perpendicular to its surface?

44. What are the three smallest non-zero thicknesses of soapy water ($n = 1.33$) on Plexiglas if it appears green (constructively reflecting 520-nm light) when illuminated perpendicularly by white light?

45. Suppose you have a lens system that is to be used primarily for 700-nm red light. What is the second thinnest coating of fluorite (magnesium fluoride) that would be nonreflective for this wavelength?

46. (a) As a soap bubble thins it becomes dark, because the path length difference becomes small compared with the wavelength of light and there is a phase shift at the top surface. If it becomes dark when the path length difference is less than one-fourth the wavelength, what is the thickest the bubble can be and appear dark at all visible wavelengths? Assume the same index of refraction as water. (b) Discuss the fragility of the film considering the thickness found.

47. To save money on making military aircraft invisible to radar, an inventor decides to coat them with a nonreflective material having an index of refraction of 1.20, which is between that of air and the surface of the plane. This, he reasons, should be much cheaper than designing Stealth bombers. (a) What thickness should the coating be to inhibit the reflection of 4.00-cm wavelength radar? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

3.5 The Michelson Interferometer

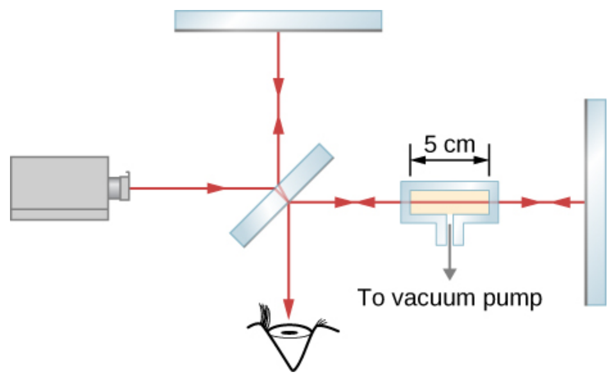
48. A Michelson interferometer has two equal arms. A mercury light of wavelength 546 nm is used for the interferometer and stable fringes are found. One of the arms is moved by $1.5\mu\text{m}$. How many fringes will cross the observing field?

49. What is the distance moved by the traveling mirror of a Michelson interferometer that corresponds to 1500 fringes passing by a point of the observation screen? Assume that the interferometer is illuminated with a 606 nm spectral line of krypton-86.

50. When the traveling mirror of a Michelson interferometer is moved 2.40×10^{-5} m, 90 fringes pass by a point on the observation screen. What is the wavelength of the light used?

51. In a Michelson interferometer, light of wavelength 632.8 nm from a He-Ne laser is used. When one of the mirrors is moved by a distance *D*, 8 fringes move past the field of view. What is the value of the distance *D*?

52. A chamber 5.0 cm long with flat, parallel windows at the ends is placed in one arm of a Michelson interferometer (see below). The light used has a wavelength of 500 nm in a vacuum. While all the air is being pumped out of the chamber, 29 fringes pass by a point on the observation screen. What is the refractive index of the air?



ADDITIONAL PROBLEMS

53. For 600-nm wavelength light and a slit separation of 0.12 mm, what are the angular positions of the first and third maxima in the double slit interference pattern?

54. If the light source in the preceding problem is changed, the angular position of the third maximum is found to be $0.57°$. What is the wavelength of light being used now?

55. Red light ($\lambda = 710.\ \text{nm}$) illuminates double slits separated by a distance $d = 0.150\ \text{mm}$. The screen and the slits are 3.00 m apart. (a) Find the distance on the screen between the central maximum and the third maximum. (b) What is the distance between the second and the fourth maxima?

56. Two sources as in phase and emit waves with $\lambda = 0.42\ \text{m}$. Determine whether constructive or destructive interference occurs at points whose distances from the two sources are (a) 0.84 and 0.42 m, (b) 0.21 and 0.42 m, (c) 1.26 and 0.42 m, (d) 1.87 and 1.45 m, (e) 0.63 and 0.84 m and (f) 1.47 and 1.26 m.

57. Two slits 4.0×10^{-6} m apart are illuminated by light of wavelength 600 nm. What is the highest order fringe in the interference pattern?

58. Suppose that the highest order fringe that can be observed is the eighth in a double-slit experiment where 550-nm wavelength light is used. What is the minimum separation of the slits?

59. The interference pattern of a He-Ne laser light ($\lambda = 632.9\ \text{nm}$) passing through two slits 0.031 mm apart is projected on a screen 10.0 m away. Determine the distance between the adjacent bright fringes.

60. Young's double-slit experiment is performed immersed in water ($n = 1.333$). The light source is a He-Ne laser, $\lambda = 632.9\ \text{nm}$ in vacuum. (a) What is the wavelength of this light in water? (b) What is the angle for the third order maximum for two slits separated by 0.100 mm.

61. A double-slit experiment is to be set up so that the bright fringes appear 1.27 cm apart on a screen 2.13 m away from the two slits. The light source was wavelength 500 nm. What should be the separation between the two slits?

62. An effect analogous to two-slit interference can occur with sound waves, instead of light. In an open field, two speakers placed 1.30 m apart are powered by a single-function generator producing sine waves at 1200-Hz frequency. A student walks along a line 12.5 m away and parallel to the line between the speakers. She hears an alternating pattern of loud and quiet, due to constructive and destructive interference. What is (a) the wavelength of this sound and (b) the distance between the central maximum and the first maximum (loud) position along this line?

63. A hydrogen gas discharge lamp emits visible light at four wavelengths, $\lambda =$ 410, 434, 486, and 656 nm. (a) If light from this lamp falls on a *N* slits separated by 0.025 mm, how far from the central maximum are the third maxima when viewed on a screen 2.0 m from the slits? (b) By what distance are the second and third maxima separated for $l = 486\ \text{nm}$?

64. Monochromatic light of frequency 5.5×10^{14} Hz falls on 10 slits separated by 0.020 mm. What is the separation between the first and third maxima on a screen that is 2.0 m from the slits?

65. Eight slits equally separated by 0.149 mm is uniformly illuminated by a monochromatic light at $\lambda = 523\text{ nm}$. What is the width of the central principal maximum on a screen 2.35 m away?

66. Eight slits equally separated by 0.149 mm is uniformly illuminated by a monochromatic light at $\lambda = 523\text{ nm}$. What is the intensity of a secondary maxima compared to that of the principal maxima?

67. A transparent film of thickness 250 nm and index of refraction of 1.40 is surrounded by air. What wavelength in a beam of white light at near-normal incidence to the film undergoes destructive interference when reflected?

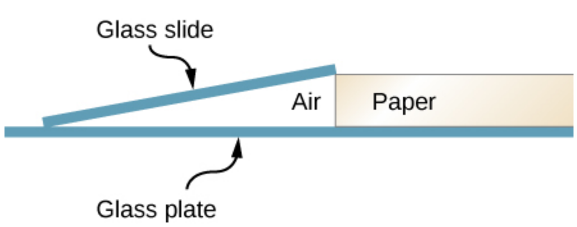
68. An intensity minimum is found for 450 nm light transmitted through a transparent film $(n = 1.20)$ in air. (a) What is minimum thickness of the film? (b) If this wavelength is the longest for which the intensity minimum occurs, what are the next three lower values of λ for which this happens?

69. A thin film with $n = 1.32$ is surrounded by air. What is the minimum thickness of this film such that the reflection of normally incident light with $\lambda = 500\text{ nm}$ is minimized?

70. Repeat your calculation of the previous problem with the thin film placed on a flat glass ($n = 1.50$) surface.

71. After a minor oil spill, a think film of oil ($n = 1.40$) of thickness 450 nm floats on the water surface in a bay. (a) What predominant color is seen by a bird flying overhead? (b) What predominant color is seen by a seal swimming underwater?

72. A microscope slide 10 cm long is separated from a glass plate at one end by a sheet of paper. As shown below, the other end of the slide is in contact with the plate. The slide is illuminated from above by light from a sodium lamp ($\lambda = 589\text{ nm}$), and 14 fringes per centimeter are seen along the slide. What is the thickness of the piece of paper?



73. Suppose that the setup of the preceding problem is immersed in an unknown liquid. If 18 fringes per centimeter are now seen along the slide, what is the index of refraction of the liquid?

74. A thin wedge filled with air is produced when two flat glass plates are placed on top of one another and a slip of paper is inserted between them at one edge. Interference fringes are observed when monochromatic light falling vertically on the plates are seen in reflection. Is the first fringe near the edge where the plates are in contact a bright fringe or a dark fringe? Explain.

75. Two identical pieces of rectangular plate glass are used to measure the thickness of a hair. The glass plates are in direct contact at one edge and a single hair is placed between them hear the opposite edge. When illuminated with a sodium lamp ($\lambda = 589\text{ nm}$), the hair is seen between the 180th and 181st dark fringes. What are the lower and upper limits on the hair's diameter?

76. Two microscope slides made of glass are illuminated by monochromatic ($\lambda = 589\text{ nm}$) light incident perpendicularly. The top slide touches the bottom slide at one end and rests on a thin copper wire at the other end, forming a wedge of air. The diameter of the copper wire is $29.45\ \mu\text{m}$. How many bright fringes are seen across these slides?

77. A good quality camera "lens" is actually a system of lenses, rather than a single lens, but a side effect is that a reflection from the surface of one lens can bounce around many times within the system, creating artifacts in the photograph. To counteract this problem, one of the lenses in such a system is coated with a thin layer of material ($n = 1.28$) on one side. The index of refraction of the lens glass is 1.68. What is the smallest thickness of the coating that reduces the reflection at 640 nm by destructive interference? (In other words, the coating's effect is to be optimized for $\lambda = 640\text{ nm}$.)

78. Constructive interference is observed from directly above an oil slick for wavelengths (in air) 440 nm and 616 nm. The index of refraction of this oil is $n = 1.54$. What is the film's minimum possible thickness?

79. A soap bubble is blown outdoors. What colors (indicate by wavelengths) of the reflected sunlight are seen enhanced? The soap bubble has index of refraction 1.36 and thickness 380 nm.

80. A Michelson interferometer with a He-Ne laser light source ($\lambda = 632.8\text{ nm}$) projects its interference pattern on a screen. If the movable mirror is caused to move by $8.54\ \mu\text{m}$, how many fringes will be observed shifting through a reference point on a screen?

81. An experimenter detects 251 fringes when the movable mirror in a Michelson interferometer is displaced. The light source used is a sodium lamp, wavelength 589 nm. By what distance did the movable mirror move?

82. A Michelson interferometer is used to measure the wavelength of light put through it. When the movable mirror is moved by exactly 0.100 mm, the number of fringes observed moving through is 316. What is the wavelength of the light?

83. A 5.08-cm-long rectangular glass chamber is inserted into one arm of a Michelson interferometer using a 633-nm light source. This chamber is initially filled with air $(n = 1.000293)$ at standard atmospheric pressure but the air is gradually pumped out using a vacuum pump until a near perfect vacuum is achieved. How many fringes are observed moving by during the transition?

84. Into one arm of a Michelson interferometer, a plastic sheet of thickness $75\ \mu\text{m}$ is inserted, which causes a shift in the interference pattern by 86 fringes. The light source has wavelength of 610 nm in air. What is the index of refraction of this plastic?

85. The thickness of an aluminum foil is measured using a Michelson interferometer that has its movable mirror mounted on a micrometer. There is a difference of 27 fringes in the observed interference pattern when the micrometer clamps down on the foil compared to when the micrometer is empty. Calculate the thickness of the foil?

86. The movable mirror of a Michelson interferometer is attached to one end of a thin metal rod of length 23.3 mm. The other end of the rod is anchored so it does not move. As the temperature of the rod changes from $15\ °\text{C}$ to $25\,\text{C}$, a change of 14 fringes is observed. The light source is a He Ne laser, $\lambda = 632.8\ \text{nm}$. What is the change in length of the metal bar, and what is its thermal expansion coefficient?

87. In a thermally stabilized lab, a Michelson interferometer is used to monitor the temperature to ensure it stays constant. The movable mirror is mounted on the end of a 1.00-m-long aluminum rod, held fixed at the other end. The light source is a He Ne laser, $\lambda = 632.8\ \text{nm}$. The resolution of this apparatus corresponds to the temperature difference when a change of just one fringe is observed. What is this temperature difference?

88. A 65-fringe shift results in a Michelson interferometer when a $42.0\text{-}\mu\text{m}$ film made of an unknown material is placed in one arm. The light source has wavelength 632.9 nm. Identify the material using the indices of refraction found in **Table 1.1**.

CHALLENGE PROBLEMS

89. Determine what happens to the double-slit interference pattern if one of the slits is covered with a thin, transparent film whose thickness is $\lambda/[2(n-1)]$, where λ is the wavelength of the incident light and n is the index of refraction of the film.

90. Fifty-one narrow slits are equally spaced and separated by 0.10 mm. The slits are illuminated by blue light of wavelength 400 nm. What is angular position of the twenty-fifth secondary maximum? What is its peak intensity in comparison with that of the primary maximum?

91. A film of oil on water will appear dark when it is very thin, because the path length difference becomes small compared with the wavelength of light and there is a phase shift at the top surface. If it becomes dark when the path length difference is less than one-fourth the wavelength, what is the thickest the oil can be and appear dark at all visible wavelengths? Oil has an index of refraction of 1.40.

92. **Figure 3.14** shows two glass slides illuminated by monochromatic light incident perpendicularly. The top slide touches the bottom slide at one end and rests on a 0.100-mm-diameter hair at the other end, forming a wedge of air. (a) How far apart are the dark bands, if the slides are 7.50 cm long and 589-nm light is used? (b) Is there any difference if the slides are made from crown or flint glass? Explain.

93. **Figure 3.14** shows two 7.50-cm-long glass slides illuminated by pure 589-nm wavelength light incident perpendicularly. The top slide touches the bottom slide at one end and rests on some debris at the other end, forming a wedge of air. How thick is the debris, if the dark bands are 1.00 mm apart?

94. A soap bubble is 100 nm thick and illuminated by white light incident at a $45°$ angle to its surface. What wavelength and color of visible light is most constructively reflected, assuming the same index of refraction as water?

95. An oil slick on water is 120 nm thick and illuminated by white light incident at a $45°$ angle to its surface. What color does the oil appear (what is the most constructively reflected wavelength), given its index of refraction is 1.40?